Last week, at the end of the Geometric Progression (GP) post, I gave you a question to figure out. I hope some of you did try it. Today we will discuss the question in detail and look at two different approaches – one without using GP formula (we discussed this approach in a previous post) and another with the formula. As I said before, you can solve every sequence question on GMAT without using the formulas we are discussing. We are still investing time in these formulas so that we can save some in the actual exam. Let me show you how.

Question 3: For every integer n from 1 to 200, inclusive, the nth term of a certain sequence is given by (-1)^n*2^(-n). If N is the sum of the first 200 terms in the sequence, then N is:

(A) less than -1
(B) between -1 and -1/2
(C) between -1/2 and 0
(D) between 0 and 1/2
(E) greater than 1/2

Solution: The question seems a little intimidating since the options give you ranges. This means that it is probably hard to find the exact value and that's just not good. Anyway, let's begin by doing what we know we should do with every sequence question if possible: we should write out the first few terms of the sequence.

First term: (-1)^1*2^(-1) = -1/2

Second term: (-1)^2*2^(-2) = 1/4

Third term: (-1)^3*2^(-3) = -1/8

and so on…

The sequence looks like this: -1/2, 1/4, -1/8, 1/16, -1/32…

N = -1/2 + 1/4 – 1/8 + 1/16 – 1/32 … (200 terms)

Method 1:

Say, we do not want to work with GPs. Let's sum the pair of consecutive terms. (Guess you are reminded of the 700+ level GMAT prep question we discussed a couple of weeks back.)

N = (-1/2 + 1/4) + (- 1/8 + 1/16) + (- 1/32 + 1/64) + …

-1/2 + 1/4 = -1/4

-1/8 + 1/16 = -1/16

-1/32 + 1/64 = -1/64

…

N = -1/4 – 1/16 – 1/64 …

We can see that the sum of all these terms will be less than -1/4 since all the rest of the terms are negative too.

N < -1/4

Let’s look at it in another way. Let’s leave the first term and sum the pair of consecutive terms thereafter.

$N = -1/2 + (1/4 – 1/8) + (1/16 – 1/32) + \dots$

$N = -1/2 + 1/8 + 1/32 + \dots$

Since all the terms after the first one are positive, N must be greater than -1/2.

Therefore, $-1/2 < N < -1/4$.

Hence, of the given ranges, N must lie between -1/2 and 0.

Answer (C)

Let’s look at a more straight forward method now.

Method 2:

Let’s say we recognize that the given sequence is a GP.

$N = -1/2 + 1/4 – 1/8 + 1/16 – 1/32 \dots$ (200 terms)

The first term is -1/2 and the common ratio is -1/2.

Sum of 200 terms of a GP = $a(1 – r^n)/(1 – r) = (-1/2)(1 – (-1/2)^{200})/(1 – (-1/2))$

Notice that in the bracket $(1 – (-1/2)^{200})$, $(-1/2)^{200}$ is a very very small number compared to 1 so the value of the bracket is approximately 1.

Sum of the GP = $(-1/2)(1)/(1- (-1/2)) = -1/3$

Since -1/3 lies between -1/2 and 0, answer is (C).

The second method was much faster and much more mechanical than the first one. I don’t particularly encourage mechanical thought process but during the exam thinking up innovative methods is a little hard if you haven’t trained your mind to do so. Hence, knowing how to deal with APs and GPs is a good idea.